4–106. Replace the force system acting on the beam by an equivalent force and couple moment at point B.

\[ F_k = \Sigma F \; ; \; F_k = 1.5 \sin 30^\circ - 2.5 \left( \frac{4}{3} \right) \]
\[ = -1.25 \; \text{kN} = 1.25 \; \text{kN} \leftarrow \]

\[ + \; \Sigma M_B ; \; M_k = -1.5 \cos 30^\circ - 2.5 \left( \frac{2}{3} \right) - 3 \]
\[ = -5.799 \; \text{kN} \cdot \text{m} = 5.799 \; \text{kN} \downarrow \]

Thus,

\[ F_k = \sqrt{F_k^2 + M_k^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \; \text{kN} \quad \text{Ans} \]

and

\[ \theta = \tan^{-1} \left( \frac{M_k}{F_k} \right) = \tan^{-1} \left( \frac{5.799}{1.25} \right) = 77.8^\circ \quad \text{Ans} \]

\[ \Sigma M_a = \Sigma M_B ; \; M_{sa} = 1.5 \cos 30^\circ (2) + 2.5 \left( \frac{2}{3} \right) (6) \]
\[ = 11.6 \; \text{kN} \cdot \text{m} \; \text{(Counterclockwise)} \quad \text{Ans} \]
4-108. Replace the two forces by an equivalent resultant force and couple moment at point O. Set \( F = 15 \text{ lb} \). 

\[ F_x = 2F_y; \quad F_x = \frac{4}{3}(15) + 20 \sin 30^\circ = 1 \text{ lb} \]

\[ F_y = 20 \cos 30^\circ + \frac{2}{3}(15) = 26.32 \text{ lb} \]

\[ F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{21 + 25.122} = 26.4 \text{ lb} \]

\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{26.32}{21} \right) = 55.7^\circ \]

\[ M_{\text{O}} = \sum M_C: \quad M_{\text{O}} = 20 \sin 30^\circ (6 \sin 40^\circ) + 20 \cos 30^\circ (3.5 + 6 \cos 40^\circ) \]

\[ = \frac{2}{3}(15)(6 \sin 40^\circ) + \frac{2}{3}(15)(3.5 + 6 \cos 40^\circ) \]

\[ = 205 \text{ lb} \cdot \text{in.} \]
4-110. Replace the force and couple moment system acting on the overhang beam by a resultant force and couple moment at point A.

**Equivalent Resultant Force:** Forces $F_1$ and $F_2$ are resolved into their $x$ and $y$ components, Fig. a. Summing these force components algebraically along the $x$ and $y$ axes,

\[ \sum (F_R)_x = 2F_a : \quad (F_R)_x = 2 \left( \frac{5}{13} \right) - 30 \sin 30^\circ = -5kN = 5kN \quad \leftarrow \]

\[ + \quad (F_R)_y = 2F_a : \quad (F_R)_y = -26 \left( \frac{12}{13} \right) - 30 \cos 30^\circ = -49.98kN = 49.98kN \quad \downarrow \]

The magnitude of the resultant force $F_R$ is given by

\[ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5^2 + 49.98^2} = 50.23kN = 50.2kN \quad \text{Ans.} \]

The angle $\theta$ of $F_R$ is

\[ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{49.98}{5} \right) = 84.29^\circ = 84.3^\circ \quad \text{Ans.} \]

**Equivalent Resultant Couple Moment:** Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

\[ \Sigma (M_R)_A = \Sigma M_A : \quad (M_R)_A = 30 \sin 30^\circ (0.3) - 30 \cos 30^\circ (2) - 26 \left( \frac{5}{13} \right) (0.3) - 26 \left( \frac{12}{13} \right) (6) - 45 \]

\[ = -239.46kN \cdot m = 239kN \cdot m \quad \text{(clockwise)} \quad \text{Ans.} \]
The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location \((x, y)\) on the slab. Take \(F_1 = 20\) kN, \(F_2 = 50\) kN.

\[
\begin{align*}
\sum F_z &= 140 \text{ kN} & \quad & \text{Ans} \\
M_{x, y} &= \sum M_y; \\
140(y) &= (50)(4) + 20(10) + 50(10) \\
&= 6.43 \text{ m} & \quad & \text{Ans} \\
M_{x, z} &= \sum M_x; \\
-140(x) &= -(50)(3) - 20(11) - 50(13) \\
&= 7.29 \text{ m} & \quad & \text{Ans}
\end{align*}
\]
Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point \( P(x, y) \) where its line of action intersects the plate.

\[
F_A = (500\text{N}) \hat{j} + 300\text{N} \hat{k} + 800\text{N} \hat{i} \quad \text{(Ans)}
\]

\[
F_A = \sqrt{(500^2 + 300^2 + 800^2)} \approx 990 \text{ N} \quad \text{(Ans)}
\]

\[
u_{xy} = (0.50311 \hat{i} + 0.3030 \hat{j} + 0.8081 \hat{k})
\]

\[
M_x = \sum M_x; \\
M_x = 800 \times (4 - y)
\]

\[
M_y = \sum M_y; \\
M_y = 800x
\]

\[
M_z = \sum M_z; \\
M_z = 500y + 300(6 - x)
\]

Since \( M_y \) also acts in the direction of \( u_{xy} \),

\[
M_y(0.50311) = 800 \times (4 - y)
\]

\[
M_y(0.3030) = 800x
\]

\[
M_y(0.8081) = 500y + 300(6 - x)
\]

\[
M_y = 3.07 \text{kN}\cdot\text{m} \quad \text{(Ans)}
\]

\[
x = 1.16 \text{ m} \quad \text{(Ans)}
\]

\[
y = 2.06 \text{ m} \quad \text{(Ans)}
\]
4-142. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.

**Loading:** The distributed loading can be divided into four parts as shown in Fig. a. The magnitude and location of the resultant force of each part acting on the beam are also indicated in Fig. a.

**Resultants:** Equating the sum of the forces along the y axis of Figs. a and b,

\[ + \downarrow F_R = \Sigma F_y; \quad F_R = \frac{1}{2}(15)(3) + \frac{1}{2}(5)(3) + 10(3) + \frac{1}{2}(10)(3) = 75 \text{ kN} \downarrow \quad \text{Ans.} \]

If we equate the moments of \( F_R \), Fig. b, to the sum of the moment of the forces in Fig. a about point A,

\[ + (M_R)_A = \Sigma M_A; \quad -75(3) = \frac{1}{2}(15)(3)(1) - \frac{1}{2}(5)(3)(1) - 10(3)(1.5) - \frac{1}{2}(10)(3)(4) \quad \overline{x} = 1.20 \text{ m} \quad \text{Ans.} \]
The beam is subjected to the distributed loading. Determine the length \( b \) of the uniform load and its position \( a \) on the beam such that the resultant force and couple moment acting on the beam are zero.

Require: \( F_b = 0 \).

\[ + F_b = \Sigma F_x ; \quad 0 = 180 - 40b \]
\[ b = 4.50 \text{ ft} \]

Ans

Require \( M_x = 0 \). Using the result \( b = 4.50 \text{ ft} \), we have

\[ \Sigma M_x = 0; \quad 0 = 180(12) - 40(4.50) \left( a + \frac{4.50}{2} \right) \]
\[ a = 9.75 \text{ ft} \]

Ans
4-156. Replace the loading by an equivalent resultant force and couple moment acting at point B.

\[ F_1 = \frac{1}{2} (4)(50) = 150 \text{ lb} \]

\[ F_2 = (4)(50) = 200 \text{ lb} \]

\[ F_3 = (6)(50) = 300 \text{ lb} \]

\[ \sum F_x = F_1 + F_2 + F_3 = 150 \sin 60^\circ + 300 \sin 60^\circ = 389.71 \text{ lb} \]

\[ \sum M_N = -M_1 + M_2 = 150 \cos 60^\circ + 300 \cos 60^\circ + 200 = 425 \text{ lb} \]

\[ F = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb} \quad \text{Ans} \]

\[ \theta = \tan^{-1} \left( \frac{425}{389.71} \right) = 47.5^\circ \quad \text{Ans} \]

\[ M_{R_{x \theta}} = \sum M_0 : \quad M_{R_{x \theta}} = 150 \cos 60^\circ (4 \cos 60^\circ + 4) + 150 \sin 60^\circ (4 \sin 60^\circ) + 300 \cos 60^\circ (3 \cos 60^\circ + 4) + 300 \sin 60^\circ (3 \sin 60^\circ) + 200 (2) \]

\[ M_{R_{x \theta}} = 2800 \text{ lb \cdot ft} = 2.80 \text{ kip \cdot ft} \quad \text{Ans} \]
4-168. Determine the magnitude of the moment of the force \( F_C \) about the hinged axis \( a-a \) of the door.

\[
r_{AB} = \left[ (-0.5 - (-0.5)) \mathbf{i} + (0 - (-1)) \mathbf{j} + (0 - 0) \mathbf{k} \right] \mathbf{m} = (1 \mathbf{j}) \mathbf{m}
\]

\[
F_C = 250 \left( \sqrt{(-0.5 - (-2.5))^2 + (0 - (-1 + 1.5 \cos 30^\circ))^2 + (0 - 1.5 \sin 30^\circ)^2} \right) \mathbf{k} \mathbf{N} = (159.33 \mathbf{i} + 183.15 \mathbf{j}) - 59.73 \mathbf{k} \mathbf{N}
\]

**Moment of Force \( F_C \) About \( a-a \) Axis**: The unit vector along the \( a-a \) axis is \( \mathbf{i} \).

Applying Eq. 4-11, we have

\[
M_{a-a} = 1 \cdot (r_{AB} \times F_C) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.73 \end{vmatrix} \mathbf{k} \mathbf{N} \cdot \mathbf{m} = 1 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - (183.15 \mathbf{i}) (0) - 0 \mathbf{N} \cdot \mathbf{m} = 59.7 \mathbf{N} \cdot \mathbf{m}
\]

The negative sign indicates that \( M_{a-a} \) is directed toward negative \( x \) axis.

\[ M_{a-a} = 59.7 \mathbf{N} \cdot \mathbf{m} \quad \text{(Ans)} \]
5-1. Draw the free-body diagram of the 50-kg paper roll which has a center of mass at \( G \) and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram. (See Fig. 5-7b.)

The Significance of Each Force:

- \( W \) is the effect of gravity (weight) on the paper roll.
- \( N_b \) and \( N_g \) are the smooth blade reactions on the paper roll.

5-7. Draw the free-body diagram of the "spanner wrench" subjected to the 20-lb force. The support at \( A \) can be considered a pin, and the surface of contact at \( B \) is smooth. Explain the significance of each force on the diagram. (See Fig. 5-7b.)

\( A_x, A_y, N_b \) force of cylinder on wrench.