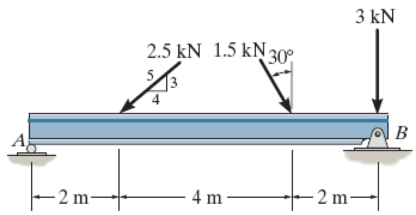
Homework Solutions # 6

4–106. Replace the force system acting on the beam by an equivalent force and couple moment at point B.

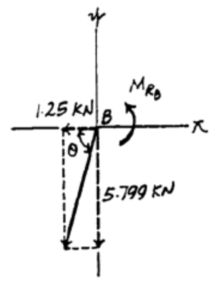


$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$

$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$

$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$

$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans}$$

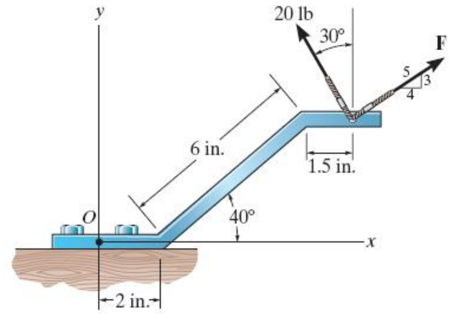
and

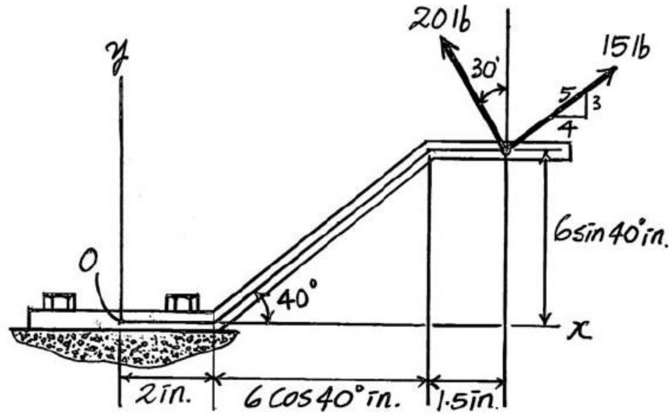
$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans}$$

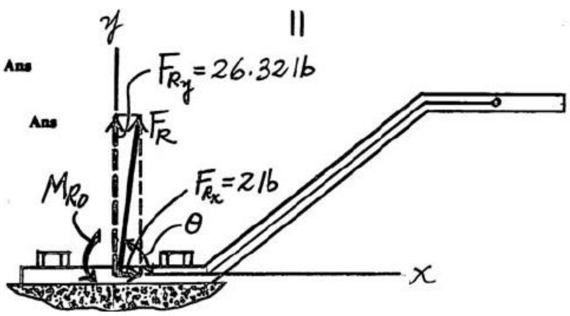
$\circlearrowleft + M_{R_B} = \Sigma M_B; \quad M_{R_B} = 1.5\cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$

$= 11.6 \text{ kN}\cdot\text{m}$ ***(Counterclockwise)*** **Ans**

*4–108. Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 15$ lb.







$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = \frac{4}{5}(15) - 20 \sin 30° = 2$ lb

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 20 \cos 30° + \frac{3}{5}(15) = 26.32$ lb

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{2^2 + 26.32^2} = 26.4 \text{ lb} \qquad \textbf{Ans}$$

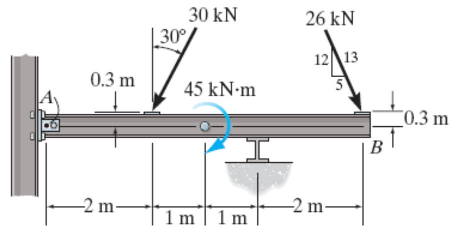
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{26.32}{2}\right) = 85.7° \qquad \textbf{Ans}$$

$\circlearrowleft + M_{R_O} = \Sigma M_O;\quad M_{R_O} = 20 \sin 30°(6 \sin 40°) + 20 \cos 30°(3.5 + 6 \cos 40°)$

$$- \frac{4}{5}(15)(6 \sin 40°) + \frac{3}{5}(15)(3.5 + 6 \cos 40°)$$

$$= 205 \text{ lb} \cdot \text{in.} \qquad \textbf{Ans}$$

4–110. Replace the force and couple moment system acting on the overhang beam by a resultant force and couple moment at point A.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5^2 + 49.98^2} = 50.23\text{ kN} = 50.2\text{ kN}$$ **Ans.**

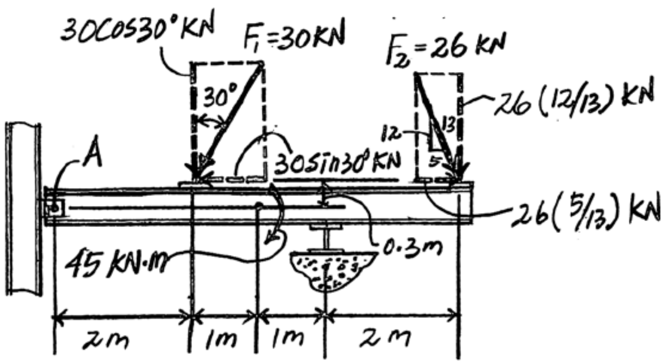
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3°$$ **Ans.**

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

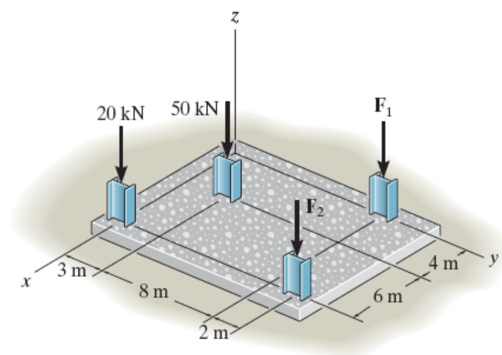
$$\circlearrowleft+(M_R)_A = \Sigma M_A; \quad (M_R)_A = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$= -239.46\text{ kN}\cdot\text{m} = 239\text{ kN}\cdot\text{m (clockwise)}$$ **Ans.**



(a)

4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



$+\downarrow F_R = \Sigma F_z;$ $F_R = 20 + 50 + 20 + 50 = 140$ kN **Ans**

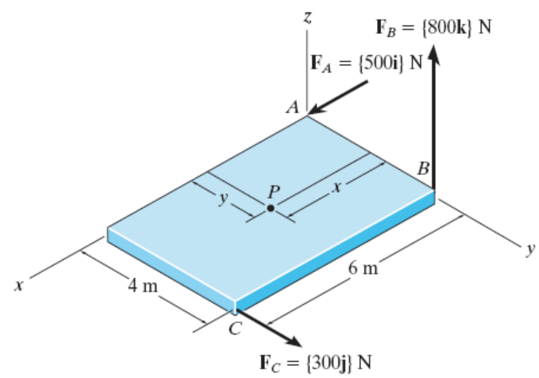
$M_{R\,y} = \Sigma M_y;$ $140(x) = (50)(4) + 20(10) + 50(10)$

$x = 6.43$ m **Ans**

$M_{R\,x} = \Sigma M_x;$ $-140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29$ m **Ans**

•**4–141.** Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



$$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}\ \text{N}$$

$$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{u}_{F_R} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$$

$$M_{R_{x'}} = \Sigma M_{x'}; \qquad M_{R_{x'}} = 800(4-y)$$

$$M_{R_{y'}} = \Sigma M_{y'}; \qquad M_{R_{y'}} = 800x$$

$$M_{R_{z'}} = \Sigma M_{z'}; \qquad M_{R_{z'}} = 500y + 300(6-x)$$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{F_R}$,

$$M_R(0.5051) = 800(4-y)$$

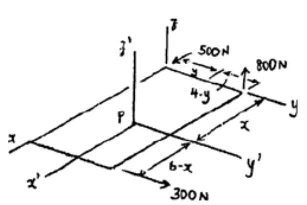
$$M_R(0.3030) = 800x$$

$$M_R(0.8081) = 500y + 300(6-x)$$

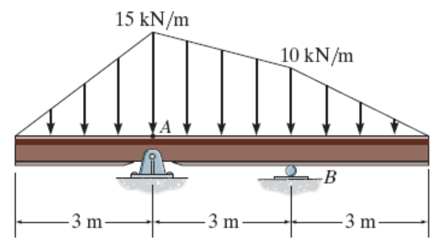
$$M_R = 3.07\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$x = 1.16\ \text{m} \qquad \textbf{Ans}$$

$$y = 2.06\ \text{m} \qquad \textbf{Ans}$$



4–142. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into four parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also indicated in Fig. *a*.

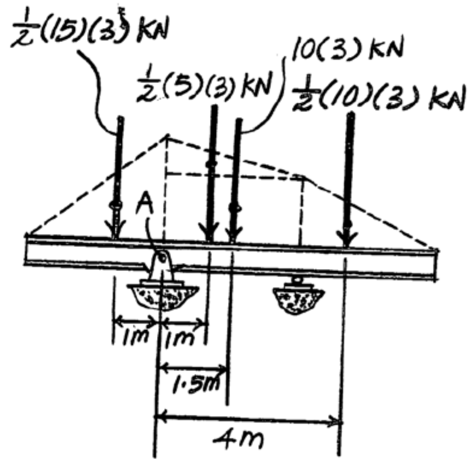
Resultants: Equating the sum of the forces along the y axis of Figs. *a* and *b*,

$+\downarrow F_R = \Sigma F_y;$ $$F_R = \frac{1}{2}(15)(3) + \frac{1}{2}(5)(3) + 10(3) + \frac{1}{2}(10)(3) = 75\text{ kN}\downarrow$$ **Ans.**

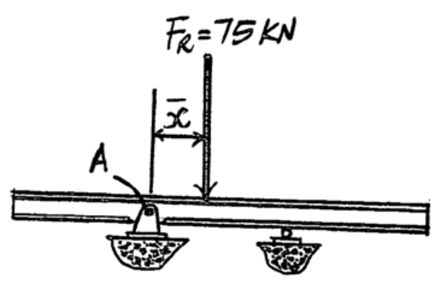
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A;\quad -75(\bar{x}) = \frac{1}{2}(15)(3)(1) - \frac{1}{2}(5)(3)(1) - 10(3)(1.5) - \frac{1}{2}(10)(3)(4)$$

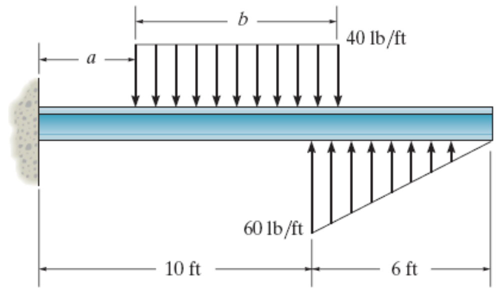
$$\bar{x} = 1.20\text{ m}$$ **Ans.**



(a)



4–150. The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

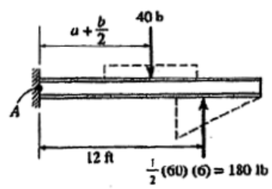
$+\uparrow F_R = \Sigma F_y;\quad 0 = 180 - 40b$

$b = 4.50$ ft **Ans**

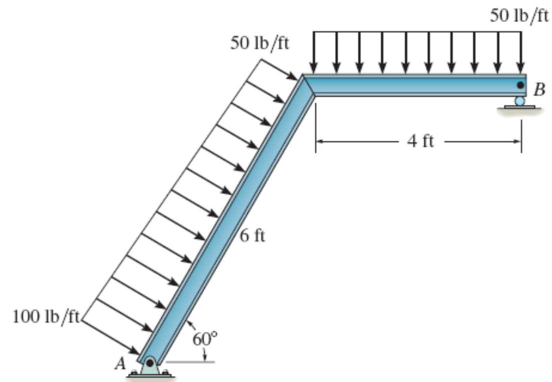
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

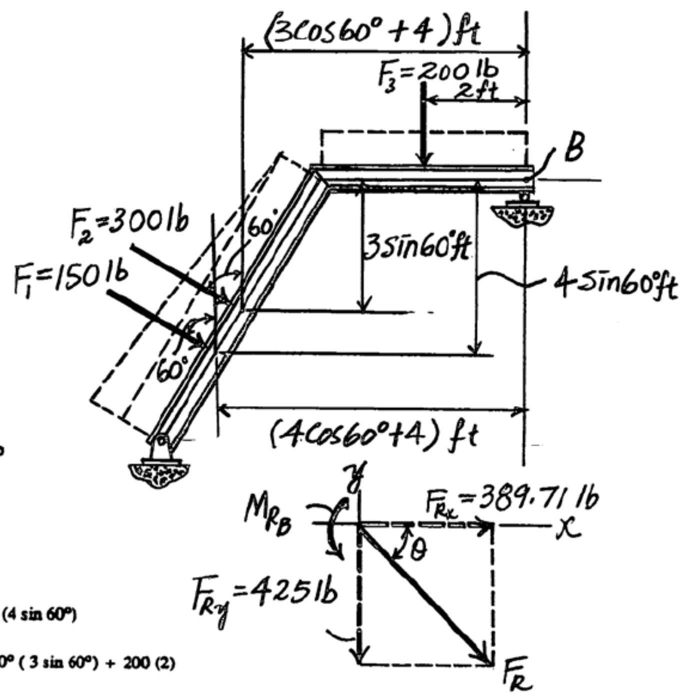
$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$a = 9.75$ ft **Ans**



*4–156.** Replace the loading by an equivalent resultant force and couple moment acting at point *B*.





$$F_1 = \frac{1}{2}(6)(50) = 150 \text{ lb}$$

$$F_2 = (6)(50) = 300 \text{ lb}$$

$$F_3 = (4)(50) = 200 \text{ lb}$$

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = 150 \sin 60° + 300 \sin 60° = 389.71 \text{ lb}$$

$$+\downarrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 150 \cos 60° + 300 \cos 60° + 200 = 425 \text{ lb}$$

$$F_R = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb} \quad \text{Ans}$$

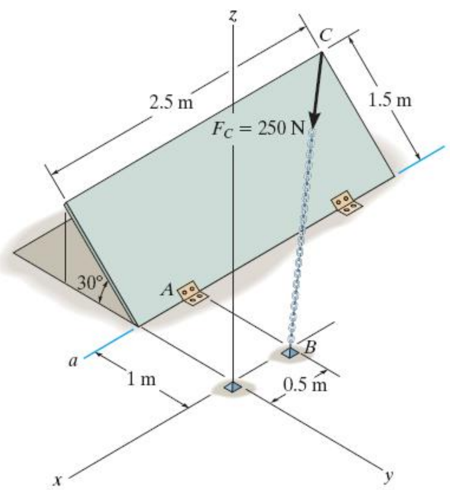
$$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5° \quad \text{Ans}$$

$$\curvearrowleft + M_{RB} = \Sigma M_B; \quad M_{RB} = 150 \cos 60° (4 \cos 60° + 4) + 150 \sin 60° (4 \sin 60°)$$

$$+ 300 \cos 60° (3 \cos 60° + 4) + 300 \sin 60° (3 \sin 60°) + 200 (2)$$

$$M_{RB} = 2800 \text{ lb} \cdot \text{ft} = 2.80 \text{ kip} \cdot \text{ft} \quad \text{Ans}$$

***4–168.** Determine the magnitude of the moment of the force $\mathbf{F}_C$ about the hinged axis *aa* of the door.



$$\mathbf{r}_{AB} = \{[-0.5-(-0.5)]\mathbf{i}+[0-(-1)]\mathbf{j}+(0-0)\mathbf{k}\}\ \text{m} = \{1\mathbf{j}\}\ \text{m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i}+\{0-[-(1+1.5\cos 30^\circ)]\}\mathbf{j}+(0-1.5\sin 30^\circ)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2+\{0-[-(1+1.5\cos 30^\circ)]\}^2+(0-1.5\sin 30^\circ)^2}}\right)\text{N}$$

$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$

Moment of Force $\mathbf{F}_C$ About a-a Axis : The unit vector along the $a-a$ axis is $\mathbf{i}$. Applying Eq. 4–11, we have

$$M_{a-a} = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F}_C)$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

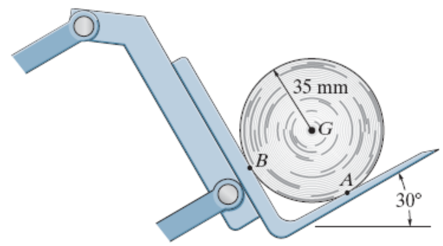
$$= 1[1(-59.75)-(183.15)(0)]-0+0$$

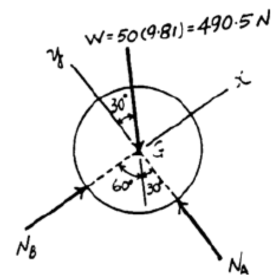
$$= -59.7\ \text{N}\cdot\text{m}$$

The negative sign indicates that $\mathbf{M}_{a-a}$ is directed toward negative x axis.

$\mathbf{M}_{a-a} = 59.7\ \text{N}\cdot\text{m}$ **Ans**

•5–1. Draw the free-body diagram of the 50-kg paper roll which has a center of mass at G and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



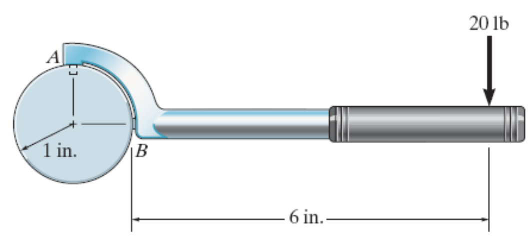


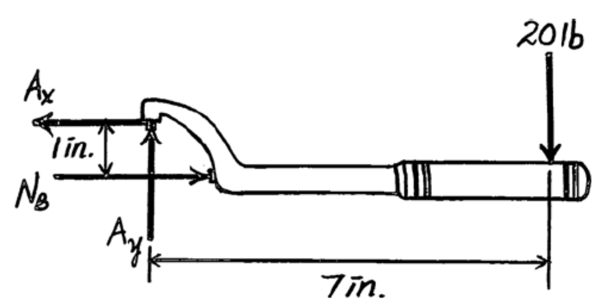
The Significance of Each Force :

W is the effect of gravity (weight) on the paper roll.

N_A and N_B are the smooth blade reactions on the paper roll.

5–7. Draw the free-body diagram of the “spanner wrench” subjected to the 20-lb force. The support at A can be considered a pin, and the surface of contact at B is smooth. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





A_x, A_y, N_B force of cylinder on wrench.