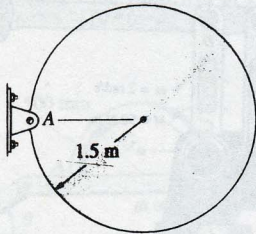**17-53.** The 80-kg disk is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 0 \qquad \textbf{Ans}$$

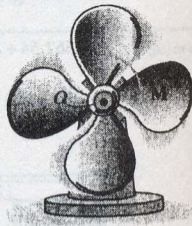$$+ \uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 80(9.81) = -80(1.5)(\alpha)$$

$$\zeta + \Sigma M_A = I_A \alpha; \qquad 80(9.81)(1.5) = [\frac{3}{2}(80)(1.5)^2]\alpha$$

$$\alpha = 4.36 \text{ rad/s}^2$$

$$A_y = 262 \text{ N} \qquad \textbf{Ans}$$

17-55. The fan blade has a mass of 2 kg and a moment of inertia $I_O = 0.18$ kg·m² about an axis passing through its center O. If it is subjected to a moment of $M = 3(1 - e^{-0.2t})$ N·m, where t is in seconds, determine its angular velocity when $t = 4$ s starting from rest.

$(\overset{\curvearrowright}{+} \Sigma M_O = I_O \alpha; \quad 3(1 - e^{-0.2t}) = 0.18\alpha$
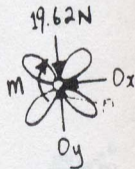
$$\alpha = 16.67(1 - e^{-0.2t})$$

$d\omega = \alpha \, dt$

$\int_0^\omega d\omega = \int_0^4 16.67(1 - e^{-0.2t}) \, dt$

$\omega = 16.67 \left[t + \frac{1}{0.2} e^{-0.2t} \right]_0^4$

$\omega = 20.8$ rad/s   **Ans**

19.62N

17-62. Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of rollers at A and B. The rollers turn with no friction.
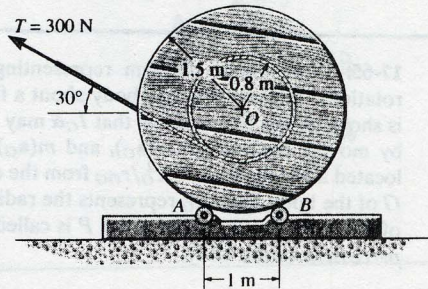
Equation of Motion : The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600\left(1.2^2\right) = 864$ kg·m^2. Applying Eq. 17–16, we have
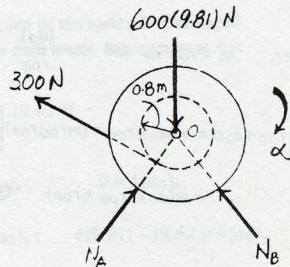
$$\left(+\ \Sigma M_O = I_O\,\alpha; \qquad -300(0.8) = -864\alpha \qquad \alpha = 0.2778\ \text{rad/s}^2\right.$$

**Kinematic :** Here, the angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2$, we have

$$(\ +) \qquad\qquad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)\,t^2$$

$$t = 6.71\ \text{s} \qquad\qquad \textbf{Ans}$$

*17-91. Two men exert constant vertical forces of 40 lb and 30 lb at the ends A and B of a uniform plank which has a weight of 50 lb. If the plank is originally at rest, determine the acceleration of its center and its angular acceleration. Assume the plank to be a slender rod.
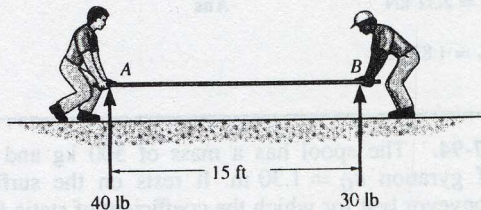

Equation of Motion : The mass moment of inertia of the plank about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{50}{32.2}\right)(15^2) = 29.115$ slug $\cdot$ ft^2. Applying Eq. 17 – 14, we have

$$+\uparrow \Sigma F_y = m(a_G)_y ; \qquad 40 + 30 - 50 = \left(\frac{50}{32.2}\right)a_G$$
$$a_G = 12.9 \text{ ft/s}^2 \qquad \textbf{Ans}$$

$$\left(+ \Sigma M_G = I_G \alpha: \qquad 30(7.5) - 40(7.5) = -29.115\alpha\right.$$
$$\alpha = 2.58 \text{ rad/s}^2 \qquad \textbf{Ans}$$

17-94. The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.
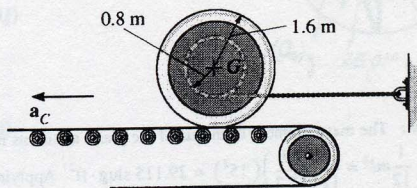
$\xrightarrow{+} \sum F_x = m(a_G)_x; \quad T - 0.5N_s = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \quad N_s - 500(9.81) = 0$

$\mathbf{\curvearrowleft} + \sum M_G = I_G\alpha, \quad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_p = a_C + a_{p/G}$

$(a_p)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

Solving;

$N_s = 4905$ N

$T = 3.13$ kN **Ans**

$\alpha = 1.684$ rad/s **Ans**

$a_G = 1.347$ m/s^2

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$a_C = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$
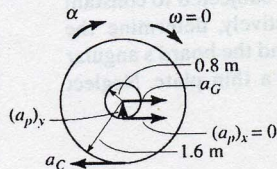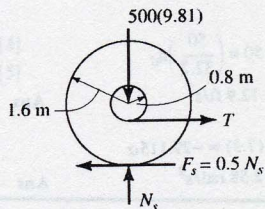
$a_C = 1.35$ m/s^2 **Ans**

Also,

$\mathbf{\curvearrowleft} + \sum M_{IC} = I_{IC}\alpha; \quad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

Since $N_s = 4905$ N

$\alpha = 1.684$ rad/s

*17-100. A uniform rod having a weight of 10 lb is pin-supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.
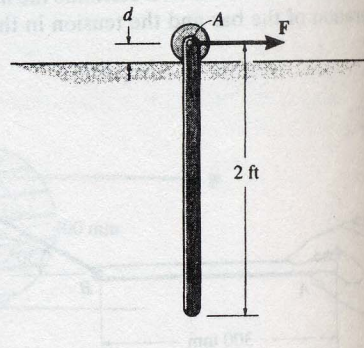
Equation of Motion : The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035$ slug · ft². At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17 – 16, we have

$$\Sigma F_t = m(a_G)_t ; \qquad 15 = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 48.3 \text{ ft/s}^2$$
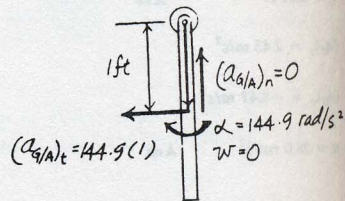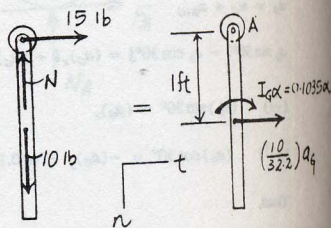
$$\left(+\Sigma M_A = \Sigma(M_k)_A ; \qquad 0 = \left(\frac{10}{32.2}\right)(48.3)(1) - 0.1035\alpha$$
$$\alpha = 144.9 \text{ rad/s}^2$$

Kinematic : Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16 – 17, we have

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\begin{bmatrix} 48.3 \\ \rightarrow \end{bmatrix} = \begin{bmatrix} a_A \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 144.9(1) \\ \leftarrow \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$(\xrightarrow{+}) \qquad 48.3 = a_A - 144.9$$
$$a_A = 193 \text{ ft/s}^2 \qquad\qquad \textbf{Ans}$$

17-102. The lawn roller has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If it is pushed forward with a force of 200 N when the handle is at 45°, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.1$, respectively.
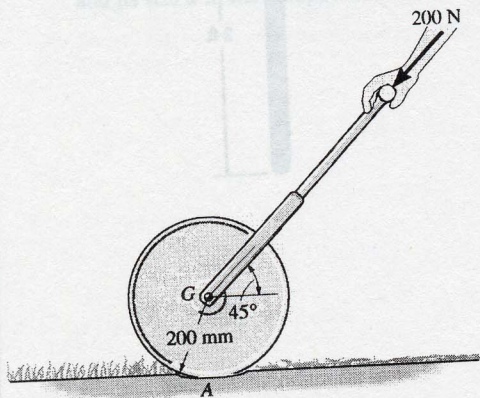
Assume no slipping.

$$I_A = I_G + md^2 = 80(0.175)^2 + 80(0.200)^2 = 5.65 \text{ kg} \cdot \text{m}^2$$

$$M_A = 200 \cos 45° (0.200) = 28.284 \text{ N} \cdot \text{m}$$

$$M_A = I_A \alpha$$

$$\alpha = \frac{28.284}{5.65} = 5.01 \text{ rad/s}^2 \quad \textbf{Ans}$$

Check no slippage assumption.

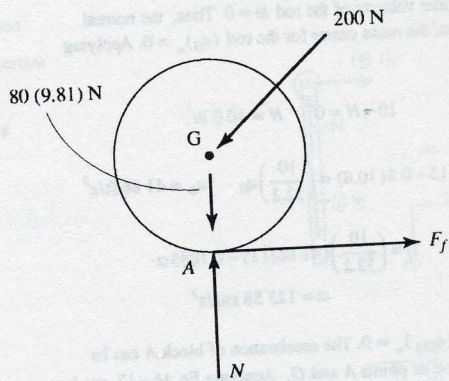$$(\uparrow +) \sum F_y = 0 \quad -200 \sin 45° - 80(9.81) + N = 0$$

$$N = 926.22 \text{ N}$$

$$(F_f)_{max} = 0.12(926.22) = 111.15 \text{ N}$$

$$I_G = 80(0.175)^2 = 2.45 \text{ kg} \cdot \text{m}^2$$

$$M_G = I_G \alpha = 2.45(5.01) = 12.27 \text{ N} \cdot \text{m}$$

$$F_f = \frac{12.27}{0.200} = 61.4 \text{ N} < 111.15 \text{ N} \quad \textbf{O.K.}$$

17-107. The 16-lb bowling ball is cast horizontally onto a lane such that initially $\omega = 0$ and its mass center has a velocity $v = 8$ ft/s. If the coefficient of kinetic friction between the lane and the ball is $\mu_k = 0.12$, determine the distance the ball travels before it rolls without slipping. For the calculation, neglect the finger holes in the ball and assume the ball has a uniform density.
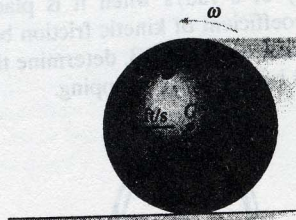
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.12 N_A = \frac{16}{32.2} a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 16 = 0$$

$$\left(+\Sigma M_G = I_G \alpha; \qquad 0.12 N_A (0.375) = \left[\frac{2}{5}\left(\frac{16}{32.2}\right)(0.375)^2\right]\alpha\right.$$
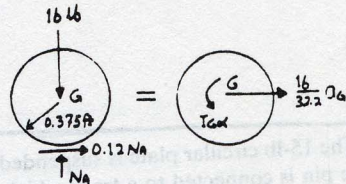
Solving,

$$N_A = 16 \text{ lb}; \qquad a_G = 3.864 \text{ ft/s}^2; \qquad \alpha = 25.76 \text{ rad/s}^2$$

When the ball rolls without slipping $v = \omega(0.375)$,

$$(\;+) \qquad \omega = \omega_0 + \alpha_c t$$

$$\frac{v}{0.375} = 0 + 25.76t$$

$$v = 9.660t$$

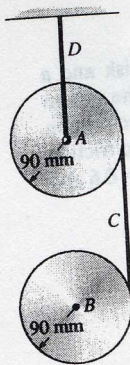$$(\xleftarrow{+}) \qquad v = v_0 + a_c t$$

$$9.660t = 8 - 3.864t$$

$$t = 0.592 \text{ s}$$

$$(\xleftarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s = 0 + 8(0.592) - \frac{1}{2}(3.864)(0.592)^2$$

$$s = 4.06 \text{ ft} \qquad \textbf{Ans}$$

17-110. A cord C is wrapped around each of the two 10-kg disks. If they are released from rest, determine the tension in the fixed cord D. Neglect the mass of the cord.

For A:

$$\uparrow + \sum M_A = I_A \alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \quad (1)$$

For B:

$$\curvearrowright + \sum M_B = I_B \alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \quad (2)$$

$$+\downarrow \sum F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \quad (3)$$

$$a_B = a_P + (a_{B/P})_t + (a_{B/P})_n$$

$$(+\downarrow)a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \quad (4)$$

Solving,

$$a_B = 7.85 \text{ m/s}^2$$
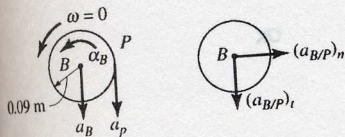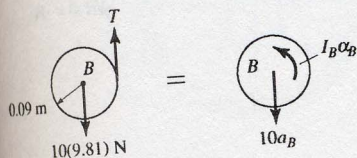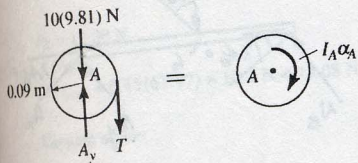
$$\alpha_A = 43.6 \text{ rad/s}^2$$

$$\alpha_B = 43.6 \text{ rad/s}^2$$

$$T = 19.62 \text{ N}$$

$$A_y = 10(9.81) + 19.62$$

$$= 118 \text{ N} \qquad \textbf{Ans}$$

18-2. The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2 \text{ N} \cdot \text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta) \text{ N} \cdot \text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.
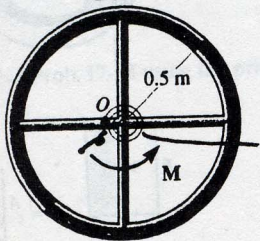
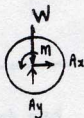$$I_0 = 2[\frac{1}{12}(2)(1)^2] + 5(0.5)^2 = 1.583$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{4\pi} 2\theta \, d\theta = \frac{1}{2}(1.583)\omega^2$$

$$(4\pi)^2 = 0.7917\omega^2$$

$$\omega = 14.1 \text{ rad/s} \qquad \textbf{Ans}$$

*18-4. The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.


$\omega = 20$ rad/s

0.5 ft   1 ft

O

B   30 lb

A   20 lb

$T = \frac{1}{2} I_O \omega_O^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2 = 283 \text{ ft} \cdot \text{lb}$ **Ans**
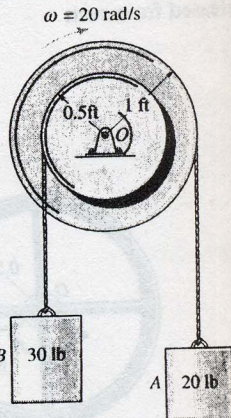
18-9. A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.
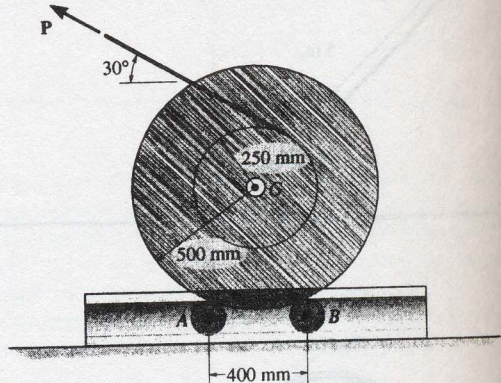
$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$

$\omega = 2.02$ rad/s   **Ans**