Contact Stresses and Deformations

ME EN 7960 – Precision Machine Design
Topic 7

Curved Surfaces in Contact

- The theoretical contact area of two spheres is a point (= 0-dimensional)
- The theoretical contact area of two parallel cylinders is a line (= 1-dimensional)
  → As a result, the pressure between two curved surfaces should be infinite
  → The infinite pressure at the contact should cause immediate yielding of both surfaces
- In reality, a small contact area is being created through elastic deformation, thereby limiting the stresses considerably
- These contact stresses are called Hertz contact stresses
Curved Surfaces in Contact – Examples

Rotary ball bearing

Linear bearings (ball and rollers)

Rotary roller bearing

Curved Surfaces in Contact – Examples (contd.)

Ball screw

Gears
Spheres in Contact

The radius of the contact area is given by:

\[ a = \sqrt{\frac{3F}{4E_1} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)} \]

Where \( E_1 \) and \( E_2 \) are the moduli of elasticity for spheres 1 and 2 and \( \nu_1 \) and \( \nu_2 \) are the Poisson’s ratios, respectively.

The maximum contact pressure at the center of the circular contact area is:

\[ p_{max} = \frac{3F}{2\pi a^2} \]

Spheres in Contact (contd.)

- The equations for two spheres in contact are also valid for:
  - Sphere on a flat plate (a flat plate is a sphere with an infinitely large radius)
  - Sphere in a spherical groove (a spherical groove is a sphere with a negative radius)
Spheres in Contact – Principal Stresses

The principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$ are generated on the z-axis:

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\text{max}} \left(1 + \nu \left(1 - \frac{Z}{a} \arctan \frac{a}{Z} \right) - \frac{1}{2 \left( \frac{Z^2}{a^2} + 1 \right)} \right)$$

$$\sigma_3 = \sigma_z = -p_{\text{max}} \left( \frac{Z^2}{a^2} + 1 \right)^{-1}$$

The principal shear stresses are found as:

$$|\tau_1| = |\tau_2| = \tau_{\text{max}} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$|\tau_3| = 0$$

Spheres in Contact – Vertical Stress Distribution at Center of Contact Area

- The maximum shear and Von Mises stress are reached below the contact area
- This causes pitting where little pieces of material break out of the surface
Cylinders in Contact

The half-width $b$ of the rectangular contact area of two parallel cylinders is found as:

$$b = \frac{4F \left[ \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]}{\pi L \left( \frac{1}{R_1} + \frac{1}{R_2} \right)}$$

Where $E_1$ and $E_2$ are the moduli of elasticity for cylinders 1 and 2 and $\nu_1$ and $\nu_2$ are the Poisson’s ratios, respectively. $L$ is the length of contact. The maximum contact pressure along the center line of the rectangular contact area is:

$$P_{\text{max}} = \frac{2F}{bL}$$

Cylinders in Contact (contd.)

- The equations for two cylinders in contact are also valid for:
  - Cylinder on a flat plate (a flat plate is a cylinder with an infinitely large radius)
  - Cylinder in a cylindrical groove (a cylindrical groove is a cylinder with a negative radius)
Cylinders in Contact – Principal Stresses

The principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$ are generated on the z-axis:

$$\sigma_1 = \sigma_2 = -2\nu p_{\text{max}} \left[ \frac{z^2}{b^2} + 1 \right] \frac{z}{b}$$

$$\sigma_2 = \sigma_3 = -p_{\text{max}} \left[ \frac{2 - \left( \frac{z^2}{b^2} + 1 \right)^{\frac{1}{2}}} {\frac{z^2}{b^2} + 1 - \frac{z}{b}} \right]$$

$$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_2 = \frac{\sigma_1 - \sigma_3}{2}, \quad \tau_3 = \frac{\sigma_1 - \sigma_2}{2}$$

Cylinders in Contact – Vertical Stress Distribution along Centerline of Contact Area

- The maximum shear and Von Mises stress are reached below the contact area.
- This causes pitting where little pieces of material break out of the surface.

Plot shows material with Poisson’s ratio $\nu = 0.3$.
Sphere vs. Cylinder – Von Mises Stress

- The Von Mises stress does not increase linearly with the contact force.
- The point contact of a sphere creates significantly larger stresses than the line contact of a cylinder.

Effects of Contact Stresses - Fatigue
Elastic Deformation of Curved Surfaces

The displacement of the centers of two spheres is given by:

\[ \delta_s = 1.04 \left[ F \left( \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right) \right]^{\frac{1}{2}} \left[ \frac{1}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{\frac{1}{2}} \]

The displacement of the centers of two cylinders is given by:

With \( v_1 = v_2 = v \), and \( E_1 = E_2 = E \):

\[ \delta_c = \frac{2F(1-v^2)}{\pi b} \left( \frac{2}{3} + \ln \frac{4R_1}{b} + \ln \frac{4R_2}{b} \right) \]

Note that the center displacements are highly nonlinear functions of the load.

Sphere vs. Cylinder – Center Displacement

- The point contact of a sphere creates significantly larger center displacements than the line contact of a cylinder.
Sphere vs. Cylinder – Stiffness

- The point contact of a sphere creates significantly lower stiffness than the line contact of a cylinder

Effects of Material Combinations

- The maximum contact pressure between two curved surfaces depends on:
  - Type of curvature (sphere vs. cylinder)
  - Radius of curvature
  - Magnitude of contact force
  - Elastic modulus and Poisson’s ratio of contact surfaces
- Through careful material pairing, contact stresses may be lowered
Contact Pressure Depending on Material Combination

• Materials with a lower modulus will experience larger deformations, resulting in a lower contact pressure.

Center Displacement Depending on Material Combination